Affine Invariant Curve Matching

by:

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Motivation

• Establish wide baseline **image correspondences** via curve matching

Motivation

Use iso-intensity level set curves.

- 😊 They are Jordan curves if they do not hit the boundaries.
- 😊 Contrast invariance property.
- 😊 If the image is defined on a continuous domain...
- 😞 ...but it is defined on a discrete lattice!
- 😞 Curves are not very smooth.
- 😞 Curves do not always have a “semantic” value.

Assumption: two corresponding curves lie on a planar surface.
Presentation Overview

- Affine Invariant Matching
- The Shape of a Curve
- The Approach
- Experimental Results
- Conclusions and Future Work
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Affine Invariant Matching

If a curve:
- lies on a plane,
- is imaged by cameras far from such plane,

Then:
- the transformation between the imaged curves can be approximated with an affine transformation.
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Normalization procedure (see also [Åström 1993]):

- Let $\Gamma$ be a Jordan curve that defines boundary of the region $\Omega$
- Let $V(\Omega) \overset{\text{def}}{=} \int_{\Omega} dx^2$ be the area of $\Omega$
- Let $m(\Omega) \overset{\text{def}}{=} \frac{1}{V(\Omega)} \int_{\Omega} x \, dx^2$ be the centroid of $\Omega$.
- Let $\Sigma(\Omega) \overset{\text{def}}{=} \frac{1}{V(\Omega)} \int_{\Omega} [x - m(\Omega)] [x - m(\Omega)]^T \, dx^2$ be the covariance of $\Omega$. 
The shape of a curve - Theory

- The shape of $\Gamma$ is defined as:
  \[
  S(\Gamma) \overset{\text{def}}{=} \left\{ s \in \mathbb{R}^2 : s = \Sigma(\Omega)^{-\frac{1}{2}} [x - m(\Omega)] \text{ for } x \in \Omega \right\}
  \]

- Let $\Gamma_1$ and $\Gamma_2$ be related by an affine transformation:
  \[
  \Gamma_2 = \{ x_2 \in \mathbb{R}^2 : \exists x_1 \in \Gamma_1 \text{ such that } x_2 = Ax_1 + b \}
  \]

Then $S(\Gamma_1)$ and $S(\Gamma_2)$ are geometrically congruent via a 2-dimensional rotation.
The Shape of a Curve - Example

\[ \Gamma_1 \]

\[ \Gamma_2 = A \Gamma_1 + b \]

\[ S(\Gamma_1) \]

\[ S(\Gamma_2) = R S(\Gamma_2) \]
The Shape of a Curve - Implementation

- Domain discretization.
  - Inside vs. Outside
- Computation ($\int \rightarrow \sum$) of:
  - Area of $\Omega$.
  - Centroid of $\Omega$.
  - Covariance of $\Omega$.
- Shape computation.
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The Approach - Overview

- Curve normalization + rotation invariant descriptors = affine invariant matching
- Curve labelling
  - Curve extraction
  - Curve filtering
  - Shape computation
  - Computation of shape rotation invariant descriptors
- Curve matching
  - Comparison of shape descriptors
The Approach - Shape Descriptors

- Variation of Goshtasby method [Goshtasby, 1985]
  - Non-uniform radial sampling
  - Quasi rotation invariant

Comparison:

\[
d(A, B) = \min_{-N_\theta+1 \leq i \leq N_\theta-1} \sum_{h=1}^{N_R} \sum_{k=1}^{N_\theta} A_{h,k} \text{ XOR } B_{h,<k-i>N_\theta}
\]
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Multiview Curve Database **MCD:**

- 40 objects (extracted from the MPEG-7 shape database) imaged under 7 different points of view.
- An arbitrary rotation and reflection applied to each original curve.
- The database consists of 14 curves for each of the 40 objects.
- Available contacting the authors: see
  
  http://vision.ece.ucsb.edu/~zuliani/MCD/MCD.shtml
Experimental Results - MCD

Precision Recall Curve

$N_R = 12, N_\theta = 15$ (updated version)
Experimental Results - Graffiti

Wide Baseline Matching example
Experimental Results - Graffiti Details

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Conclusion and Future work

Conclusions
- A definition of shape of Jordan curves has been proposed.
- The normalization procedure can be used with any rotation invariant curve descriptor.

Future work
- The normalization procedure should be faster.
- Curve location in the scene should be integrated in the matching process.
The End

Thanks for your attention.
Precision Recall Performance

- Precision-recall curve is used to measure the descriptor performance.
  - \( A(\Gamma, T) \) is the set of \( T \) retrievals (based on the smallest distances from \( \Gamma \) in the descriptor space)
  - \( R(\Gamma) \) is the set of 14 images in the dataset relevant to \( \Gamma \).

- **Precision**: \( P(\Gamma, T) \overset{\text{def}}{=} \frac{|A(\Gamma, T) \cap R(\Gamma)|}{T} \)
  - Proportion of items retrieved that are relevant.

- **Recall**: \( C(\Gamma, T) \overset{\text{def}}{=} \frac{|A(\Gamma, T) \cap R(\Gamma)|}{14} \)
  - Proportion of relevant items that are retrieved.

- The precision recall curve is plotted by averaging precision and recall over all \( \Gamma \).
Bibliography Extension


